

10105

Reg. No.: \_\_\_\_\_

Name: \_\_\_\_\_

**APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY**

FIRST SEMESTER B.TECH DEGREE SPECIAL EXAMINATION, AUGUST 2016

**Course Code: MA101****Course Name: CALCULUS**

Max. Marks: 100

Duration: 3 Hours

**PART A***Answer ALL questions. Each question carries 3 marks*

1. Find derivative of  $y = \sinh(4x-8)$
2. Test whether the series converges or diverges,  $\sum_{k=1}^{\infty} \frac{k}{2^k}$
3. Identify the surface  $z = y^2 - x^2$
4. Convert from rectangular to spherical co-ordinates,  $(2\sqrt{3}, 2, -4)$
5. Find  $\frac{\partial Z}{\partial x}$  and  $\frac{\partial Z}{\partial y}$  if  $Z = \cos(xy^3)$
6. Show that  $\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial^2 z}{\partial y \partial x}$  if  $z = x^2y + 5y^3$ .
7. Evaluate  $\int_0^2 (2t\hat{i} + 3t^2\hat{j}) dt$
8. Find the arc length of the parametric curve  $x=e^t, y=e^{-t}, z=\sqrt{2}t, 0 \leq t \leq 1$ .
9. Evaluate  $\int_1^3 \int_2^4 (40 - 20xy) dy dx$
10. Evaluate  $\int_0^3 \int_0^2 \int_0^1 (xyz) dx dy dz$

**PART B***Answer any 2 complete questions each having 7 marks*

11. Test the convergence of the series  $\sum_{k=1}^{\infty} \frac{k(k+3)}{(k+1)(k+2)(k+5)}$
12. Show that  $\sinh^{-1} x = \ln(x + \sqrt{x^2 + 1})$
13. Find the Taylor series of  $\frac{1}{x+2}$  about  $x=1$ .

*Answer any 2 complete questions each having 7 marks*

14. Express the equation  $x^2 - y^2 - z^2 = 0$  in cylindrical and spherical coordinates.

15. Evaluate  $\lim_{(x,y) \rightarrow (0,0)} [\sin(\sqrt{x^2 + y^2})]/(x^2 + y^2)$  by converting to polar coordinates.
16. Show that the functions  $f(x, y) = 3x^2y^5$  and  $f(x, y) = \sin(3x^2y^5)$  are continuous everywhere.

**Answer any 2 complete questions each having 7 marks**

17. Let  $L(x, y)$  denote the local linear approximation to  $f(x, y) = \sqrt{x^2 + y^2}$  at the point  $(3, 4)$ . Compare the error in approximating  $f(3.04, 3.98) = \sqrt{(3.04)^2 + (3.98)^2}$  by  $L(3.04, 3.98)$  with the distance between the points  $(3, 4)$  and  $(3.04, 3.98)$ .
18. Suppose that  $w = x^2 + y^2 - z^2$  and  $x = \rho \sin \phi \cos \theta$ ,  $y = \rho \sin \phi \sin \theta$ ,  $z = \rho \cos \phi$ . Use appropriate form of the chain rule to find  $\frac{\partial w}{\partial \rho}$  and  $\frac{\partial w}{\partial \theta}$ .
19. Locate the relative extrema and saddle points of  $f(x, y) = 3x^2 - 2xy + y^2 - 8y$ .

**Answer any 2 complete questions each having 7 marks**

20. Let  $f(x, y) = x^2 e^y$ . Find the maximum value of a directional derivative at  $(-2, 0)$  and find the unit vector in the direction in which the maximum value occur.
21. Find the angle between the tangent lines to the graphs of  $r_1(t) = \tan^{-1} t \mathbf{i} + \sin t \mathbf{j} + t^2 \mathbf{k}$   
 $r_2(t) = (t^2 - t) \mathbf{i} + (2t - 2) \mathbf{j} + \log t \mathbf{k}$
22. Suppose that a particle moves through 3-space so that its position vector at time  $t$  is  $\mathbf{r}(t) = t \mathbf{i} + t^2 \mathbf{j} + t^3 \mathbf{k}$ . Find the scalar tangential and normal components of acceleration at time  $t = 1$ .

**Answer any 2 complete questions each having 7 marks**

23. Use a polar double integral to find the area enclosed by the circle  $r = \sin \theta$
24. Use a triple integral to find the volume of the solid within the cylinder  $x^2 + y^2 = 9$  and between the planes  $z = 1$  and  $z = 5$
25. Evaluate  $\iint_R \frac{x-y}{x+y} dA$  where  $R$  is the region enclosed by  $x - y = 0$ ,  $x - y = 1$ ,  $x + y = 1$ ,  
 $x + y = 3$