

Reg.No.....

Name.....

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY**FIRST SEMESTER B.TECH DEGREE (SUPPLEMENTARY) EXAMINATION,
FEBRUARY 2017 (2015 ADMISSION)**Course Code: **MA 101**Course Name: **CALCULUS**

Max.Marks : 100

Duration : 3 Hours

PART A**(Answer all questions. Each question carries 3 marks)**

- 1) Show that the series $\sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^n$ converges.
- 2) Classify the surface $z = (x - 1)^2 + (y + 2)^2 + 3$
- 3) Find the Maclaurin series for $\cos x$
- 4) Evaluate $\lim_{(x,y) \rightarrow (-1,2)} \frac{xy}{x^2 + y^2}$
- 5) Convert the cylindrical co-ordinate into rectangular co ordinate of $(4, \pi/3 - 3)$.
- 6) Find the slope of the surface $z = xy^2$ in the x direction at the point $(2,3)$.
- 7) Find the directional derivative of $f = x^2y - yz^3 + z$ at $(1,-2,0)$ in the direction of $\vec{a} = 2\vec{i} + \vec{j} + 2\vec{k}$
- 8) Find the unit normal to the surface $xy + xz + yz = c$ at $(-1,2,3)$
- 9) Evaluate $\int_1^a \int_1^b x^2 y \, dx dy$
- 10) Find the area of the region R enclosed by $y = 1, y = 2, x = 0, x = y$.

PART B**(Answer any 2 questions. Each question carries 7 marks)**

- 11) Test the absolute convergence of $\sum_{n=1}^{\infty} \frac{(-1)^n n^4}{4^n}$
- 12) Determine the Taylor's series expansion of $f(x) = \sin x$ at $x = \pi/2$.
- 13) Test the convergence of $\frac{1}{1.2.3} + \frac{3}{2.3.4} + \frac{5}{3.4.5} + \dots$

(Answer any 2 questions. Each question carries 7 marks)

- 14) Find the equation of the paraboloid $z = x^2 + y^2$ in the cylindrical and spherical coordinates.
- 15) Find $F(f(x), g(y), h(z))$ if $F(x, y, z) = y e^{xyz}$, $f(x) = x^2$, $g(y) = y + 1$, $h(z) = 2z^2$
- 16) By converting into polar coordinate evaluate $\lim_{(x,y) \rightarrow (0,0)} \sqrt{x^2 + y^2} \ln \left((x^2 + y^2)^2 \right)$

(Answer any 2 questions. Each question carries 7 marks)

- 17) Find the local linear approximation L of $f(x, y, z) = xyz$ at the point $P(1, 2, 3)$. Compare the error in approximating f by L at the point $Q(1.001, 2.002, 3.003)$ with the distance PQ .
- 18) Find the relative extrema of $f(x, y) = 3x^2 - 2xy + y^2 - 8y$
- 19) If f is a differentiable function of three variables and suppose that

$$w = f(x - y, y - z, z - x) \quad \text{Show that} \quad \frac{\partial w}{\partial x} + \frac{\partial w}{\partial y} + \frac{\partial w}{\partial z} = 0$$

(Answer any 2 questions. Each question carries 7 marks)

- 20) Suppose that a particle moves along a curve in 3-space so that its position vector at time t is $r(t) = 4\cos \pi t \mathbf{i} + 4\sin \pi t \mathbf{j} + t \mathbf{k}$. Find the distance travelled and the displacement of the particle during the time interval $1 \leq t \leq 5$
- 21) A particle is moving along the curve, $\vec{r} = (t^3 - 2t)\vec{i} + (t^2 - 4)\vec{j}$ where t denotes the time. Find the scalar tangential and normal components of acceleration at $t = 1$. Also find the vector tangential and normal components of acceleration at $t = 0$.
- 22) Find the arc length of the parametric curve $x = 5\cos t$, $y = 5\sin t$, $z = 2t$; $0 \leq t \leq \pi$

(Answer any 2 questions. Each question carries 7 marks)

- 23) Evaluate the integral by converting into polar co ordinates $\int_0^{2\sqrt{4-x^2}} \int_0^x (x^2 + y^2) dy dx$
- 24) Using triple integral to find the volume bounded by the cylinder $x^2 + y^2 = 4$ and the planes $z = 0$ and $y + z = 3$
- 25) Change the order of integration and evaluate $\int_0^1 \int_x^1 \frac{x}{x^2 + y^2} dx dy$
